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Structural Identification Using Inverse System Dynamics

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A time-domain technique is presented that uses inverse structural dynamics to identify physical characteristics of a structure that can subsequently be used for damage detection. The term inverse refers to the roles of input and output being reversed from the usual forward system structural dynamics problem. If sensors such as accelerometers are placed at the external input locations, modal parameters corresponding to structural motion with the sensor locations fixed can be identified. One of the main advantages of this approach is that it only requires measured response data. This characteristic makes the approach applicable in cases where the input forces cannot be measured.

I. Introduction

THE motivation for the development of the structural identification technique presented in this paper is damage detection. Currently, modal identification is a very popular approach to detecting structural damage. Usually, pre- and postdamage vibration experiments are performed and damage is detected by the observation of changes in frequencies and mode shapes. Reference 1 presents a very comprehensive review of modal-based damage detection techniques.

In many situations, however, typical vibration tests are not easily performed while the structure is in service. In such cases, the structural characteristics may have to be extracted from response due to unknown ambient excitation. A method, called the natural excitation technique (NExT), was developed at Sandia National Laboratories[2] specifically to handle such problems. The approach is based on the computation of the cross-correlation function between each output channel and a set of reference outputs selected analogously to the placement of actuators for a typical modal test. The authors have shown that the cross-correlation function of two responses can be expressed as a sum of decaying sinusoids with the same characteristics as the structure's unit pulse response. Therefore, the cross-correlation functions can be used as pulse response functions in time-domain modal parameter estimation techniques such as the eigensystem realization algorithm (ERA).[3,4] The difficulty associated with applying NExT in many cases is that the technique assumes the input forces are random and stationary.

The identification method presented here is based on inverse system dynamics. This approach is new in that it does not attempt to identify the usual mode shapes and frequencies of a structure. Instead it identifies the vibrational characteristics of an inverse representation of the structural system being considered. The inverse representation of a system differs from the usual forward representation in that the roles of the input and output are reversed. The inverse vibrational characteristics to be identified consist of frequencies and mode shapes for the inverse system. In some cases, they represent quantities as intimately related to the forward system as constrained modal parameters. In this situation, inverse frequencies and mode shapes can be easily related to structural parameters and characteristics. Changes in the inverse modal parameters can then be related to corresponding changes in the structure.

The main advantages of the use of the inverse system identification approach are that the force inputs to the structure do not have to be known and there are no requirements for the input forces to be random or stationary. An example of current interest is damage detection in large space structures (LSS) such as Russia's Mir space station and the yet to be constructed International Space Station. An obvious source of excitation in these examples is the act of docking with the shuttle. The unknown docking forces exciting the LSS are neither random nor stationary.

II. Theory

The systems considered in this work can be represented by the state-space equation

$$\dot{x} = Ax + Bu \tag{1}$$

in which x represents an n dimensional state vector, A is the $n \times n$ system matrix, B is the $n \times n_a$ input influence matrix, and u is the n_a dimensional force vector. The corresponding system output equation is given by

$$y_s = C_s x + D_s u \tag{2}$$

where y_s is an n_s dimensional sensor output vector, C_s is an $n_s \times n$ output influence matrix, and D_s is an $n_s \times n_a$ direct transmission matrix.

It is assumed throughout this analysis that accelerometers are used as sensors such that the direct transmission matrix is nonzero; however, other types of sensors can be used by appropriately modifying the derivations using lagged systems. It is also assumed that the structure being considered is free–free. For the case of a modal representation given by

$$\ddot{q} + 2\varsigma\omega\dot{q} + \omega^2 q = \phi_a^T u \tag{3}$$

the matrices A and B have the form

$$A = \begin{bmatrix} 0 & I \\ -\omega^2 & -2\varsigma\omega \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ \phi_a^T \end{bmatrix} \tag{4}$$

whereas matrices C_s and D_s are given by

$$C_s = \begin{bmatrix} -\phi_s\omega^2 & -2\phi_s\varsigma\omega \end{bmatrix} \qquad D_s = \phi_s\phi_a^T \tag{5}$$

in which ω is a diagonal matrix of modal frequencies, ς is a diagonal matrix of modal damping coefficients, and ϕ_s and ϕ_a are the mode shapes partitioned to the sensor and input locations, respectively.

A. Formulation of Remote Sensing System

Kammer[5] previously developed a technique for measuring the response of an operating structure at discrete locations and for predicting the response at other desired locations on the structure that do not have sensors during operation. It was shown that an input–output representation called a remote sensing system (RSS) can be derived directly from vibration test data where the input is the measured response at the sensor locations and the output is the desired response at remote locations. Note that to identify the RSS, the system response at both the sensor locations and the desired locations must be measured.

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An analytical representation of the RSS can be derived by solving Eq. (2) for the force input at time t

$$\boldsymbol{u}(t) = -D_s^+ C_s \boldsymbol{x}(t) + D_s^+ \boldsymbol{y}_s(t) \tag{6}$$

in which D_s^+ represents the generalized inverse $D_s^+ = (D_s^T D_s)^{-1} D_s^T$. Note that the number of sensors n_s must be greater than or equal to the number of inputs n_a. Equation (6) is then substituted into the state equation (1) producing

$$\dot{\boldsymbol{x}} = \hat{A}\boldsymbol{x} + \hat{B}\boldsymbol{y}_s \tag{7}$$

where the new plant and input influence matrices are given by

$$\hat{A} = A - BD_s^+ C_s \qquad \hat{B} = BD_s^+ \tag{8}$$

Equations (6) and (7) represent an input–output system in which the roles of input and output have been reversed with respect to the usual forward system represented by Eqs. (1) and (2). Equations (6) and (7), therefore, represent an inverse system.

The forward system output equation for the response at the desired remote locations is analogous to Eq. (2)

$$\boldsymbol{y}_d = C_d \boldsymbol{x} + D_d \boldsymbol{u} \tag{9}$$

where

$$C_d = \begin{bmatrix} -\phi_d \omega^2 & -2\phi_d \varsigma \omega \end{bmatrix} \qquad D_d = \phi_d \phi_a^T \tag{10}$$

and ϕ_d are the mode shapes partitioned to the desired locations. Finally, the equation for the input force (6) can be substituted into Eq. (9) to produce

$$\boldsymbol{y}_d = \hat{C}\boldsymbol{x} + \hat{D}\boldsymbol{y}_s \tag{11}$$

with

$$\hat{C} = C_d - D_d D_s^+ C_s \qquad \hat{D} = D_d D_s^+ \tag{12}$$

Equations (7) and (11) represent the RSS in which measured response is the input and desired response is the output. The input force acting on the structure has been completely eliminated from the system equations. Note that the state equations for both the inverse system and the RSS are identical.

All real systems operate in continuous time and are sampled discretely, which results in a discrete-time representation. In discrete time, the RSS can be represented by the convolution equation

$$\boldsymbol{y}_d(k) = \sum_{i=0}^{k} P_i \boldsymbol{y}_s(k - i) \tag{13}$$

in which the $n_d \times n_s$ weighting matrices P_i are the RSS Markov parameters or unit pulse response, and k represents the appropriate time step. The Markov parameters represent the free-decay response of the RSS. Thus, a modal identification technique, such as ERA, can be used to identify the corresponding RSS mode shapes and frequencies. Equations (7), (8), (11), and (12) show that the RSS and its modal parameters depend directly on the physical parameters of the actual structure and can be used to detect structural damage based on changes in the RSS frequencies and mode shapes. However, at this point, based on Eqs. (8) and (12), the desired relationship appears to be very complicated for the general case. Specific cases of interest will be addressed in the following section.

B. Inverse System Eigenvalues and Modes

It is important to determine the physical significance of the RSS or inverse system eigenvalues such that they can be used in damage detection. The physical interpretation is dependent on the relationship between the numbers and locations of forward system inputs and outputs. Many cases can be considered, but only a few significant to modal identification are discussed in this paper.

For the special case where there are as many sensors as force inputs, $n_s = n_a$, the inverse system eigenvalues can be directly related to the forward system transmission zeros, which are defined to be the complex values ψ at which it is possible to apply a nonzero input and get an identically zero output at the sensor locations for suitable initial conditions $x(0)$. Therefore, if the input is assumed in the form $\boldsymbol{u} = \mu e^{\psi t}$, then ψ is said to be a transmission zero of the forward system (1) and (2) if

$$\begin{bmatrix} A - \psi I & B \\ -C_s & -D_s \end{bmatrix} \begin{Bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{\theta} \end{Bmatrix} = Q \begin{Bmatrix} \boldsymbol{x}(0) \\ \boldsymbol{\theta} \end{Bmatrix} = 0 \tag{14}$$

For a nontrivial solution to exist, the determinant of Q must vanish. In the case where D_s is full rank, $|Q|$ can be written as[6]

$$|Q| = |D_s| \left| A - BD_s^{-1} C_s - \psi I \right| = |D_s| |\hat{A} - \psi I| \tag{15}$$

that produces the condition

$$|\hat{A} - \psi I| = 0 \tag{16}$$

corresponding to the characteristic equation of the inverse system, which implies that ψ is also an eigenvalue of the inverse system. Reference 7 can be consulted for more details on transmission zeros and structures.

1. *Noncollocated Inputs and Outputs*

Within the particular case of equal numbers of inputs and outputs, two of the many possible subcases are considered. The first subcase is where the inputs and outputs are totally noncollocated, which means that there are no sensors at any of the input locations. In this situation, the forward system is in general nonminimum phase.[8,9] This means that some of the transmission zeros of the continuous time forward system are in the right half-plane, or for the discrete-time system, outside the unit circle. This implies that the inverse system has unstable poles. The RSS Markov parameters in Eq. (13) grow without bound. The remaining stable inverse system eigenvalues occur in complex conjugate pairs. It will be shown by example in a later section that the proposed RSS identification technique estimates the minimum phase portion of the RSS pulse response that corresponds to the stable transmission zeros and related shapes. These inverse system frequencies, or forward system transmission zeros, and their corresponding shapes can be identified from the estimated pulse response by the use of ERA. Researchers are currently developing techniques that use the forward system transmission zeros in damage detection and finite element model updating.[10]

2. *Collocated Inputs and Outputs*

Although the physical interpretation of inverse system eigenvalues as transmission zeros may be very natural to control engineers and the researchers mentioned, it has had less physical meaning to the average structural dynamicist trying to perform damage detection. This subsection describes yet another physical interpretation that is useful. The special case of collocated sensors and inputs is considered in which every force input location and direction has a corresponding sensor. In addition, it is assumed that the sensors are just sufficient in number and location to remove all rigid-body modes from the structure if constrained.

The structural dynamics community has long used the Craig–Bampton substructure representation in component mode synthesis and subsequent loads analysis.[11] A Craig–Bampton (CB) representation can be generated for the free–free forward structure with respect to the sensor locations. The usual continuous forward system structural equation of motion can be partitioned according to sensor and unmeasured degrees of freedom (DOF) as

$$\begin{bmatrix} M_{uu} & M_{us} \\ M_{su} & M_{ss} \end{bmatrix} \begin{Bmatrix} \ddot{z}_u \\ \ddot{z}_s \end{Bmatrix} + \begin{bmatrix} G_{uu} & G_{us} \\ G_{su} & G_{ss} \end{bmatrix} \begin{Bmatrix} \dot{z}_u \\ \dot{z}_s \end{Bmatrix} + \begin{bmatrix} K_{uu} & K_{us} \\ K_{su} & K_{ss} \end{bmatrix} \begin{Bmatrix} z_u \\ z_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ u \end{Bmatrix} \tag{17}$$

in which z_u is the displacement vector at the unmeasured locations and z_s is the displacement vector at the sensor locations. Note that the forces $\boldsymbol{u}$ are only applied at the sensor locations.

Equation (17) can be transformed to CB coordinate space by the use of the relation

$$z = Tz_{\mathrm{CB}} = \begin{bmatrix} \phi & \psi \\ 0 & I \end{bmatrix} \begin{Bmatrix} \boldsymbol{q} \\ z_s \end{Bmatrix} \tag{18}$$

where ϕ are the mode shapes of the system with the sensor locations fixed, q are the corresponding modal coordinates, $\psi = -K_{uu}^{-1}K_{us}$, and I is an n_s dimensional identity matrix. Note that for the present case $[\psi^T \quad I]^T = \phi_r$, the rigid-body modes of the structure. The CB displacement vector z_{CB} consists of fixed sensor location modal coordinates and the physical displacement at the sensors. Applying the transformation to Eq. (17) and then premultiplying by T^T, the CB representation produces

$$\begin{bmatrix} I & M_{qs} \\ M_{sq} & M_r \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{q}}_u \\ \ddot{z}_s \end{Bmatrix} + \begin{bmatrix} G_q & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{\boldsymbol{q}} \\ \dot{z}_s \end{Bmatrix} + \begin{bmatrix} \lambda & 0 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \boldsymbol{q} \\ z_s \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{u} \end{Bmatrix} \tag{19}$$

in which $M_{qs} = \phi^T M_{uu}\psi + \phi^T M_{us}$, M_r is the structure's rigid-body mass matrix, G_q is the modal damping matrix, and λ is a diagonal matrix of fixed mode frequencies squared.

Matrix Eq. (19) can be expanded to produce the two subequations

$$\ddot{\boldsymbol{q}} + G_q\dot{\boldsymbol{q}} + \lambda\boldsymbol{q} = -M_{qs}\boldsymbol{y}_s \tag{20}$$

$$M_r\ddot{z}_s + M_{sq}\ddot{\boldsymbol{q}} = \boldsymbol{u} \tag{21}$$

where acceleration has been assumed for sensor output, $\boldsymbol{y}_s = \ddot{z}_s$. Solving Eq. (20) for $\ddot{q}$ and substituting into Eq. (21) produces

$$\boldsymbol{u} = -M_{sq}G_q\dot{\boldsymbol{q}} - M_{sq}\lambda\boldsymbol{q} + (M_r - M_{sq}M_{qs})\boldsymbol{y}_s \tag{22}$$

Equation (20) can be written in state-space form as

$$\begin{Bmatrix} \dot{\boldsymbol{q}} \\ \ddot{\boldsymbol{q}} \end{Bmatrix} = \begin{bmatrix} 0 & I \\ -\lambda & -G_q \end{bmatrix} \begin{Bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{Bmatrix} + \begin{bmatrix} 0 \\ -M_{qs} \end{bmatrix} \boldsymbol{y}_s \tag{23}$$

with Eq. (22) becoming the output equation

$$\boldsymbol{u} = [-M_{sq}\lambda \quad -M_{sq}G_q] \begin{Bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{Bmatrix} + (M_r - M_{sq}M_{qs})\boldsymbol{y}_s \tag{24}$$

Equations (23) and (24) show that the CB representation of the structure is equivalent to an inverse structural system in which the acceleration at the sensor locations is the input to a fixed mode state equation and the output is the force applied at the sensor locations.

The RSS output equation can be formed by using Eq. (18) to recover the acceleration at the desired response locations

$$\ddot{z}_d = \phi_d\ddot{\boldsymbol{q}} + \psi_d\ddot{z}_s \tag{25}$$

where the subscript d indicates a partitioning of the associated matrix to the appropriate rows. Our substituting for $\ddot{q}$ from the state equation yields the RSS output equation as

$$\boldsymbol{y}_d = [-\phi_d\lambda \quad -\phi_d G_q] \begin{Bmatrix} \boldsymbol{q} \\ \dot{\boldsymbol{q}} \end{Bmatrix} + (\psi_d - \phi_d M_{qs})\boldsymbol{y}_s \tag{26}$$

Equations (23) and (25) give a continuous time state-space representation of the RSS that is analogous to Eqs. (7) and (11). This derivation illustrates that for the rigid-body sufficient collocated case, the RSS eigenvalues and mode shapes correspond to the frequencies and mode shapes of the structure with the sensor locations fixed.

This relationship has been discussed previously by the control dynamics community,[12] but only for single input/single output systems in which the transmission zeros correspond to the zeros of the transfer function. In general, they are not the same quantities. In the structural dynamics community, Yun Li and Craig[13] suggested that the equality between transmission zeros and inverse system eigenvalues was valid for multi-input/multi-output systems. They used a simple beam example to prove their point. This paper proves the validity of the relationship by the use of the familiar CB substructure representation and its identification as an inverse system. The result that inverse system modal parameters correspond to fixed interface modes is very important in that many techniques exist for detecting damage or changes in structures based on corresponding changes in the structure's modal parameters.

C. Identification of RSS Modal Parameters

To identify the RSS modal parameters, the RSS pulse response must first be estimated by the use of experimental data. The available experimental data are split into the two sets discussed earlier, the sensor locations numbering n_s and the desired locations numbering n_d. If there are n_e experiments, the RSS convolution equation (13) can be expanded into the matrix equation

$$PY_s = Y_d \tag{27}$$

in which

$$P = \begin{bmatrix} P_0 & P_1 & \cdots & P_{N_R-1} \end{bmatrix} \qquad Y_d = \begin{bmatrix} y_{d0} & y_{d1} & \cdots & y_{dn_t-1} \end{bmatrix}$$

$$Y_s = \begin{bmatrix} y_{s0} & y_{s1} & \cdots & \cdots & y_{sn_t-1} \\ 0 & y_{s0} & \cdots & \cdots & y_{sn_t-1} \\ 0 & 0 & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & y_{s0} & \cdots & y_{sn_t-N_R} \end{bmatrix}$$

where y_{si} is an $n_s \times n_e$ matrix in which the jth column represents the measured sensor response at time step i for experiment j, y_{di} is the corresponding response at the desired locations, and n_t is the number of experimental data points. In an application, the data matrices in Y_s and Y_d are truncated to N_C block columns. Two cases important to RSS modal identification are considered.

In the first case, there are equal numbers of sensors and experiments, and the data blocks y_{si} are assumed to be square and full rank. The corresponding data matrix Y_s theoretically should then be full row rank. If equal numbers of block rows and block columns are used, that is, if $N_R = N_C$, Y_s is square and the first N_R RSS Markov parameters can be determined uniquely by inverting Y_s in Eq. (27). In practice, the data matrix Y_s becomes very large and ill conditioned. The usual matrix inverse must be replaced by a Moore–Penrose pseudoinverse based on singular value decomposition.[4] In addition, for computational stability, the problem is made to be overconstrained by the addition of block columns such that $N_C > N_R$. The solution for the RSS pulse response is then given by

$$P = Y_d Y_s^T \left(Y_s Y_s^T\right)^+ \tag{28}$$

where the superscript + denotes the pseudoinverse. The normal form of the least-squares solution was used during this work at the risk of inaccuracy to minimize the memory requirements for the pseudoinversion of the data matrix. Alternative solution techniques, both new and existing, will be explored for improved efficiency and accuracy in future research. However, the least-squares approach used here is the standard technique used in modal identification.[4] Once the RSS pulse response is identified, ERA can be used to identify the corresponding discrete system modal parameters, which can then be transofrmed back to continuous time.

In the original development and application of the RSS technique,[5] the identification of the RSS model was based on the forward system pulse response measured during a vibration experiment. In this case, the number of experiments is equal to the number of inputs, $n_e = n_a$. The idea was to compress the system information contained in the RSS Markov parameters in the pulse response sequence into a much smaller number of equivalent terms or weighting matrices W_i. This was accomplished by using several more sensors than experiments and many more block columns than block rows. It is believed that this compression is analogous to that produced by the observer formulation approach introduced in Refs. 14 and 15. The only requirement is that the product $N_R \times n_s$ must be greater than the number of observable-controllable modes responding in the data. Because of the relatively small number of block rows used, the computation of the system equivalent RSS weighting matrices using Eq. (28) was found to be very robust to data errors and noise.

Unfortunately, when there are more sensors than experiments, the estimated weighting sequence representation of the system cannot be directly used as free-decay data in ERA to estimate the RSS modal parameters. The weighting sequence must be uncompressed into the RSS pulse response analogous to the technique used by Juang et al.,[14] who employed observer Markov parameters. The observer formulation was applied to the RSS problem during this project, but the recovery of the RSS pulse response from the observer Markov parameters was found to be unstable for inverse systems.

D. Step-by-Step Procedure for RSS Identification

The RSS identification procedure can by summarized by the following steps:

1) Measure acceleration response at selected locations on structure during excitation.

2) Split sensor measurements into the desired (d) and sensed (s) location sets.

3) Form data matrices Y_s and Y_d in Eq. (27). The number of block columns N_C in Y_s should be greater than the number of block rows N_R. Several identification analyses should be performed using varying values of N_C until convergence is achieved.

4) Compute the RSS unit pulse response P using Eq. (28).

5) Use pulse response in ERA to estimate discrete time inverse system modal parameters. The parameters are then transformed back to continuous time by the use of $\psi_{\text{continuous}} = \log(\psi_{\text{discrete}})/\Delta t$ where Δt is the data sampling time.

III. Numerical Examples

Two numerical examples will be presented to illustrate the use of RSS to identify an inverse system and its modal parameters, which can subsequently be used to detect changes in a structure that may indicate damage.

A. Docking Simulation

The first example, pictured in Fig. 1, simulates the docking between two simple elastic structures. DOF 1–7 represent a structure such as the Mir space station, whereas DOF 8 and 9 would represent the space shuttle. The physical parameters of the system are given by $m_1 = 25$, $m_2 = 50$, $m_t = 200$, $k_1 = 10^6$, $k_2 = 2\times 10^6$, $k_t = 6\times 10^5$, $c_1 = 500$, and $c_2 = 300$. To approximate the nonlinear interaction between the shuttle and Mir, the docking mechanism is represented by a cubic nonlinear spring with stiffness coefficient $k_r = 10{,}000$ and linear viscous damping coefficient $c_r = 500$. The system considered therefore comprises two linear subsystems that are nonlinearly coupled. For reference, the coupled system undamped natural frequencies where the docking spring has been made linear are listed in Table 1.

Docking is simulated by giving the shuttle masses 8 and 9 initial velocities of 40 in the negative direction. The system response is simulated using a time step of 0.002 s and the acceleration response is measured at DOF 6 and 7 on the Mir structure. The measured acceleration data is divided into the two earlier described sets for RSS identification. DOF 7 is assumed to be the sensor location, whereas DOF 6 is the desired response location. DOF 7 is also the location at which the nonlinear docking force is applied to the Mir substructure. Relative to the Mir, the system is totally collocated and the number of experiments n_e and number of sensors n_s are equal. According to

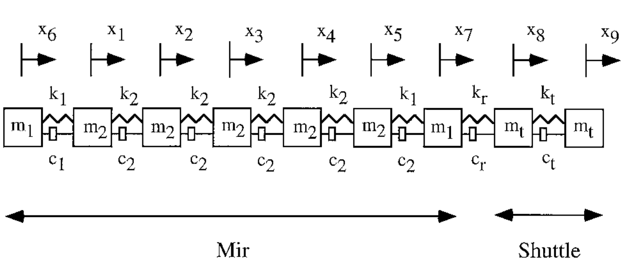

Fig. 1 Spring-mass-damper docking simulation model.

Table 1 Natural frequencies for simple docking example

Mode	Coupled system frequency, Hz	Fixed Mir frequency, Hz
1	0.00	7.16
2	1.20	21.48
3	12.35	33.59
4	16.15	43.49
5	28.77	53.63
6	37.10	61.02
7	45.03	——
8	54.11	——
9	61.11	——

the theory presented, the corresponding RSS should comprise Mir substructure modes with the sensor location fixed. The corresponding Mir undamped sensor fixed modal frequencies were computed by the use of the analytical model and are also listed in Table 1. Note that, except for the last mode, the fixed interface frequencies are quite distinct from the linear coupled system frequencies.

The technique discussed in Sec. II.C was used to estimate the RSS pulse response. A sensor location response data matrix was generated using 1000 rows and 2000 columns. The RSS pulse response was then estimated using Eq. (28) and is illustrated in Fig. 2. The corresponding frequency response function is shown in Fig. 3 with peaks representing the first five sensor fixed mode shapes clearly visible. Application of ERA to the estimated pulse response produced estimates of the fixed mode frequencies given by 7.13, 21.05, 33.46, 43.38, and 54.05 Hz, which correspond very well with the values listed in Table 1. Note that the mode at 61.02 Hz is not identified because it is not observable from the sensor at the desired location DOF 6. This example illustrates that the proposed RSS technique properly identifies the fixed sensor dynamics of the Mir substructure based only on the response of the nonlinearly coupled system.

B. Two Noncollocated Inputs and Outputs

The second example consists of a uniform free–free beam that is 3.7 m in length with a mass of 1.6 kg. The cross section is square with area 1.61×10^{-4} m^2, moment of inertia 2.17×10^{-9} m^4, elastic modulus 6.90×10^{10} N/m^2, and Poisson ratio 0.33. There is a concentrated mass of 1.2 kg at the center of the beam. A finite element representation of the beam containing 22 grid points is illustrated in Fig. 4. The beam is constrained to planar motion such that each grid point possesses a single transverse displacement and a single in-plane rotation. The finite element method (FEM) was statically reduced to the 22 transverse displacement DOF. Two rigid-body modes and the first five elastic modes with frequencies up to 60.0 Hz were retained for the simulation. A 5% modal damping was assumed, and a time step of 0.002 s was employed.

In this example, only the acceleration response of the beam due to a set of theoretically unknown forces was simulated. Two force inputs were located at DOF 10 and 22. Two sensor locations were used at DOF 8 and 19, and the desired location for response prediction was chosen as DOF 1. Two independent experiments were simulated with input forces, each consisting of five sinusoids with randomly selected frequencies between 0.0 and 60.0 Hz. In this example, the inputs and sensors are totally noncollocated, but the numbers of sensors and inputs are equal. Therefore, although the inverse system eigenvalues, listed in Table 2, do not correspond to fixed sensor mode shapes as in the collocated case, they do correspond to the forward system transmission zeros. Note that four of the eigenvalues correspond to the two rigid-body modes and eight of the eigenvalues occur in stable complex conjugate pairs. The final two eigenvalues are real, with one appearing in the right half-plane, which indicates that the forward system is nonminimum phase and the corresponding inverse system is unstable. The undamped frequencies corresponding to the stable eigenvalue pairs are also listed in Table 2.

An RSS representation was computed by the use of a data matrix with 400 block rows and 1500 block columns resulting in an

Table 2 Inverse system eigenvalues and undamped frequencies for beam example

Number	Inverse system eigenvalue Real	Inverse system eigenvalue Imaginary	Stable undamped frequency, Hz
1	0	——	2.99
2	0	——	8.59
3	0	——	25.94
4	0	——	32.63
5	4.6456*e*+02	——	——
6	−4.0130*e*+02	——	——
7	−7.3587*e*+00	+2.0486*e*+02*i*	——
8	−7.3587*e*+00	−2.0486*e*+02*i*	——
9	−9.7186*e*+00	+1.6271*e*+02*i*	——
10	−9.7186*e*+00	−1.6271*e*+02*i*	——
11	−1.5708*e*+00	+5.3963*e*+01*i*	——
12	−1.5708*e*+00	−5.3963*e*+01*i*	——
13	−7.4635*e*−01	+1.8774*e*+01*i*	——
14	−7.4635*e*−01	−1.8774*e*+01*i*	——

800 by 3000 matrix. The proposed computation method produced an RSS pulse response that included high-frequency computational noise at approximately 150.0 Hz. The computed pulse response was then low-pass filtered with a cutoff frequency of 70.0 Hz. Figure 5 illustrates the resulting filtered RSS pulse response from DOF 8 to DOF 1 and the corresponding frequency response is presented in Fig. 6. Peaks associated with three of the stable inverse system eigenvalue pairs are clearly visible. Modal parameters were then extracted from the filtered pulse response data using ERA, which produces the undamped frequencies 2.97, 8.57, 25.57, and 32.95 Hz that closely approximate the frequencies corresponding to the stable inverse system eigenvalue pairs.

The identified RSS frequencies and corresponding shapes can then be used to detect changes or damage in the structure. However, their relation to the physical parameters of the system is not as straightforward as the totally collocated case where the identified frequencies and shapes correspond to sensor fixed structural modal parameters.

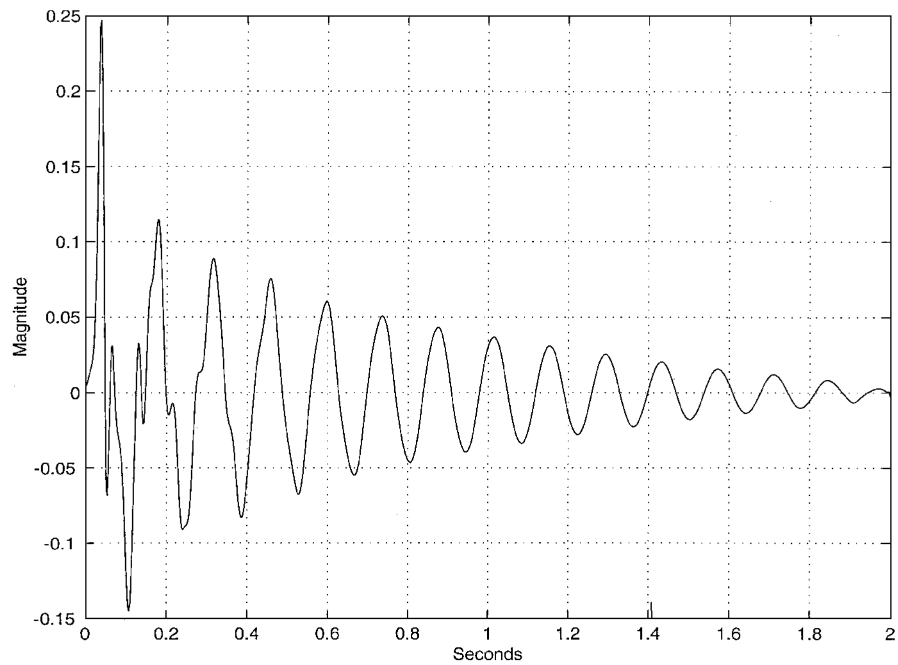

Fig. 2 RSS unit pulse response for simulated docking example from DOF 7 to DOF 6.

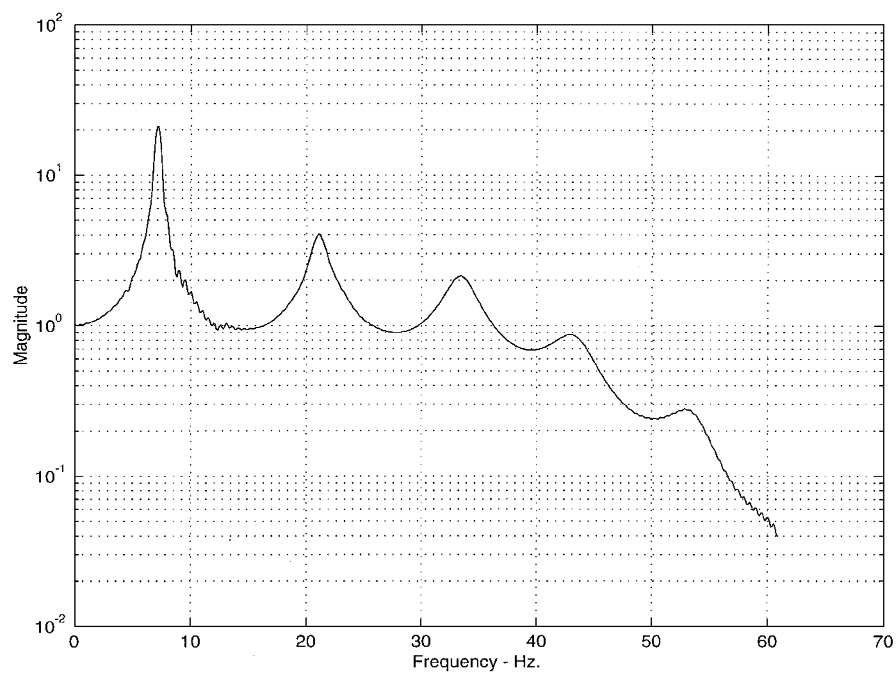

Fig. 3 Frequency response function for simulated docking example from DOF 7 to DOF 6.

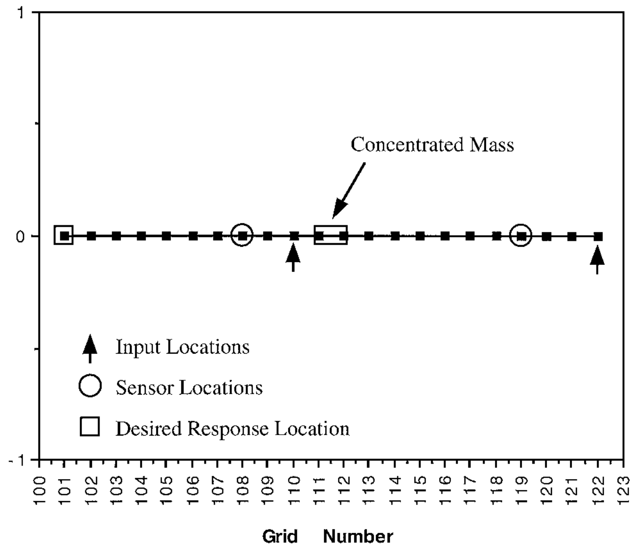

Fig. 4 Beam example FEM.

IV. Conclusion

A time-domain technique has been presented that uses inverse structural dynamics to identify physical characteristics of a structure that can subsequently be used for damage detection or finite element model updating. The term inverse refers to the roles of input and output being reversed from the usual forward system structural dynamics problem. If sensors such as accelerometers are placed at the external input locations, modal parameters corresponding to structural motion with the sensor locations fixed can be identified. The same approach can be used to identify fixed interface modes of a substructure in which sensors are placed at the interface to the complete system. If sensors are not collocated with the inputs, other important structural characteristics can be identified, such as transmission zeros and their corresponding shapes. One of the main advantages of this approach is that it only requires measured response data. The applied external forces or internal interface forces are not required for modal identification. Another important advantage of RSS is that the technique does not assume the inputs are random and stationary. These characteristics make the RSS approach applicable in cases where the deterministic and nonstationary input forces cannot be measured, such as International Space Station/shuttle docking events. It is believed that RSS provides a valuable new method to

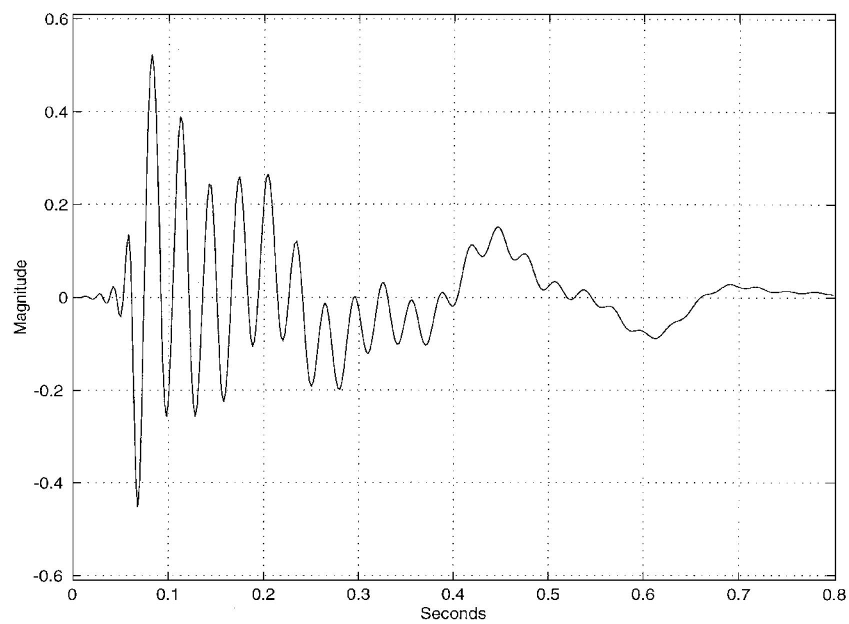

Fig. 5 Filtered beam RSS pulse response from DOF 8 to DOF 1.

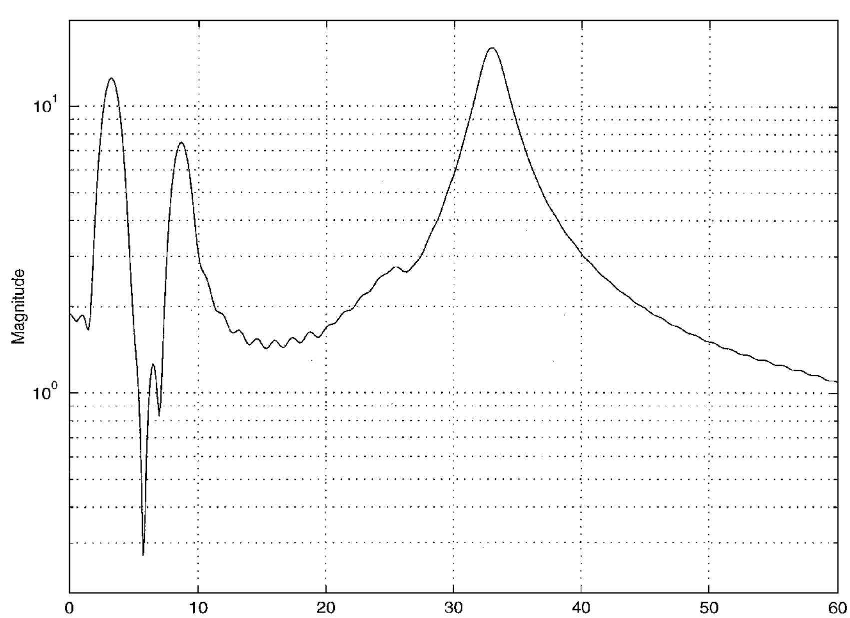

Fig. 6 RSS frequency response from DOF 8 to DOF 1.

identifying characteristics of structures from measured data without the need for measuring the input.

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